

### ABSTRACT

We consider the mathematical formulation of time varying wireless networks, where the users of the networks are in relative motion. We showed that in this case if the solution for the power of the system exist, the solution is uniformly asymptotically stable. It is also showed that the stability is global this means that for all initial conditions have same asymptotic behavior.

**KEYWORDS:** mathematical formulation of time, stability theory and wireless networks.

### INTRODUCTION

If, for the  $n$  –th order differential equation  $\frac{d^n x}{dt^n} = F\left(x, \frac{dx}{dt}, \dots, \frac{d^{n-1}x}{dt^{n-1}}; t\right)$  (1) we define the new set of variables  $x_1 = x, x_2 = \frac{dx}{dt}, \dots, x_n = \frac{d^{n-1}x}{dt^{n-1}}$  then the one  $n$  –th order differential equation with independent variable  $t$  and one dependent variable  $x$  can be replaced by the system

$$\begin{aligned} \frac{dx_1}{dt} &= x_2(t) \\ \frac{dx_2}{dt} &= x_3(t) \\ &\vdots \\ \frac{dx_{n-1}}{dt} &= x_n(t) \\ \frac{dx_n}{dt} &= F(x_1, x_2, \dots, x_n, t) \end{aligned} \quad (2)$$

of  $n$  first-order equations with independent variable  $t$  and  $n$  dependent variables  $x_1, \dots, x_n$ . In fact this is just a special case of

$$\begin{aligned} \frac{dx_1}{dt} &= F_1(x_1, x_2, \dots, x_n, t) \\ \frac{dx_2}{dt} &= F_2(x_1, x_2, \dots, x_n, t) \\ &\vdots \\ \frac{dx_{n-1}}{dt} &= F_{n-1}(x_1, x_2, \dots, x_n, t) \\ \frac{dx_n}{dt} &= F_n(x_1, x_2, \dots, x_n, t) \end{aligned} \quad (3)$$

where the right-hand sides of all the equations are now functions of the variables  $x_1, x_2, \dots, x_n$ . The system defined by (3) is called an  $n$  –th order dynamical system. Such a system is said to be autonomous if none of the functions  $F_e$  is an explicit function of  $t$ .

Picard's theorem generalizes in the natural way to this  $n$ -variable case as does also the procedure for obtained approximations to a solution with Picard iterates. That is, with the initial condition  $x(\tau) = \xi_e, e = 1, 2, \dots, n$ , we define the set of sequences  $\{X_e^{(j)}(t)\}, e = 1, 2, \dots, n$  with

$$\begin{aligned} X_e^{(0)}(t) &= \xi_e \\ X_e^{(j+1)}(t) &= \xi_e + \int_{\tau}^t F_e(X_e^{(j)}(u), \dots, X_n^{(j)}(u); u) du, \quad j = 1, 2, \dots \end{aligned}$$

For all  $e = 1, 2, \dots, n$

**Example1:** Consider the simple harmonic differential equation  $\ddot{x}(t) = -\omega^2 x(t)$  with the initial conditions  $x(0) = 0$  and  $\dot{X}(0) = \omega$ . This equation is equivalent to the system

$$\begin{aligned} \dot{X}_1(t) &= x_2(t), \quad \dot{X}_2(t) = -\omega^2 x_1(t), \\ x_1(0) &= 0, \quad x_2(0) = \omega \end{aligned}$$

which is a second-order autonomous system. From (3)

$$\begin{aligned} X_1^{(0)}(t) &= 0, & X_1^{(0)}(t) &= \omega, \\ X_1^{(1)}(t) &= 0 + \int_0^t \omega du, & X_2^{(1)}(t) &= \omega + \int_0^t \omega du, \\ &= \omega t, & &= \omega, \\ X_1^{(2)}(t) &= 0 + \int_0^t \omega du, & X_2^{(2)}(t) &= \omega + \int_0^t \omega^3 u du, \\ &= \omega t, & &= \omega \left\{ 1 - \frac{(\omega t)^2}{2!} \right\}, \\ X_1^{(3)}(t) &= 0 + \int_0^t \omega \left\{ 1 - \frac{(\omega t)^2}{2!} \right\} du, & X_2^{(3)}(t) &= \omega - \int_0^t \omega^3 u du, \\ &= \omega t - \frac{(\omega t)^3}{3!}, & &= \omega \left\{ \frac{(\omega t)^2}{2!} \right\}. \end{aligned}$$

The pattern which is emerging is clear

$$\begin{aligned} x_1^{(2j-1)}(t) = x_1^{(2j)}(t) &= \omega t - \frac{(\omega t)^3}{3!} + \dots + (-1)^{j+1} \frac{(\omega t)^{(2j-1)}}{(2j-1)!} \\ &= 1, 2, \dots \\ X_2^{(2j)}(t) = x_1^{(2j+1)}(t) &= \omega \left\{ 1 - \frac{(\omega t)^2}{2!} + \dots + (-1)^j \frac{(\omega t)^{(2j)}}{(2j)!} \right\} \end{aligned}$$

**STABILITY THEORY**

In mathematics, stability theory addresses the stability of solutions of differential equations and of trajectories of dynamical systems under small perturbations of initial conditions. The heat equation, for example,

**Definition (1):**

$$\text{Let } \dot{x} = f(x, t) \tag{4}$$

is a system of ODE,  $x = X(t)$  it is a solution is said to be:

- stable if, given any  $\epsilon > 0$  and any  $t_0 \geq 0$ , there exists a  $\delta = \delta(\epsilon, t_0)$  such that  $|x(t_0) - X(t_0)| < \delta \Rightarrow |x(t) - X(t)| < \epsilon, \forall t \geq t_0 \geq 0$ , (5)

for any solution  $x(t)$  of (1),

- uniformly stable if, for every  $\epsilon > 0$ , there exists  $\delta = \delta(\epsilon)$ , independent of  $t_0$ , such that (5) is satisfied for all  $t_0 \geq 0$ ,

- unstable if it is not stable,

- asymptotically stable if it is stable and for any  $t_0 \geq 0$  there exists a positive constant  $c = c(t_0)$  such that

$$|x(t_0) - X(t_0)| < c \Rightarrow x(t) - X(t) \rightarrow 0 \text{ as } t \rightarrow \infty,$$

for any solution  $x(t)$  of (4),

- uniformly asymptotically stable if it is uniformly stable and there exists a positive constant  $c$ , independent of  $t_0$ , such that, for every  $\eta > 0$ , there exists  $T = T(\eta) > 0$  such that, for all  $t_0 \geq 0$

$$|x(t_0) - X(t_0)| < c \Rightarrow |x(t) - X(t)| < \eta, \forall t \geq t_0 + T(\eta),$$

for any solution  $x(t)$  of (1),

- globally uniformly asymptotically stable if it is uniformly stable with  $\delta(\epsilon)$  satisfying  $\lim_{\epsilon \rightarrow \infty} \delta(\epsilon) = \infty$ ,

and, for all positive  $\eta$  and  $c$ , there exists

$$T = T(\eta, c) > 0 \text{ such that, for all } t_0 \geq 0$$

$$|x(t_0) - X(t_0)| < c \Rightarrow |x(t) - X(t)| < \eta, \forall t \geq t_0 + T(\eta, c),$$

for any solution  $x(t)$  of (4).

**LYAPUNOV STABILITY THEORY**

Various types of stability may be discussed for the solutions of differential equations or difference equations describing dynamical systems. The most important type is that concerning the stability of solutions near to a point of equilibrium. This may be discussed by the theory of Lyapunov. In simple terms, if the solutions that start out near an equilibrium point  $x_i$  stay near  $x_i$  forever, then  $x_i$  is Lyapunov stable. More strongly, if  $x_i$  is Lyapunov stable and all solutions that 'start out near  $x_i$  converge to  $x_i$ , then  $x_i$  is asymptotically stable. The notion of exponential stability guarantees a minimal rate of decay, i.e., an estimate of how quickly the solutions converge. The idea of Lyapunov stability can be extended to infinite-dimensional manifolds, where it is known as structural stability,

which concerns the behavior of different but "nearby" solutions to differential equations. Input-to-state stability (ISS) applies Lyapunov notions to systems with inputs

### linear time invariant system (LTI):

The simplest form of system  $\dot{x} = f(t, x)$  is linear, time invariant system

$$\dot{x} = Ax \quad (6)$$

Where  $A$  is a constant,  $n \times n$  matrix, the system then takes the form of a homogenous first order ODE, and, therefore, may be solved explicitly. The solution of (6). With initial state  $x(0) = x_0$ , is given by  $x(t) = \exp(At)x_0$

### Nonlinear autonomous systems:

A natural next step in the analysis of systems of the form (4) is to continue to require that the right-hand side have no explicit time-dependence, but to allow  $f$  to be a more general nonlinear functional

$$\dot{x} = f(x) \quad (7)$$

### Lyapunov's Direct Method:

Let  $V : D \rightarrow R$  be a continuously differentiable function defined on the domain  $D \subset R^n$  that contains the origin. The rate of change of  $V$  along the trajectories of (4) is given by

$$\dot{V}(x) = \frac{d}{dx} V(x) = \sum_{i=1}^n \frac{\partial V}{\partial x_i} \frac{dx_i}{dt} = \left[ \frac{\partial V}{\partial x_1} \frac{\partial V}{\partial x_2} \dots \frac{\partial V}{\partial x_n} \right] \dot{x} = \frac{\partial V}{\partial x} f(x) \quad (8)$$

The main idea of Lyapunov's theory is that if  $\dot{V}(x)$  is negative along the trajectories of the system, then  $V(x)$  will decrease as time goes forward. Moreover, we do not really need to solve the nonlinear ODE (1) for every initial condition, but we just need some information about the drift  $f(x)$ .

### Example2:

Consider the nonlinear system  $\dot{x} = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = \begin{bmatrix} -x_1 + 2x_1^2x_2 \\ -x_2 \end{bmatrix}$  candidate Lyapunov function  $V(x) = \lambda_1 x_1^2 + \lambda_2 x_2^2$ , with  $\lambda_1, \lambda_2 > 0$ . If we plot the function  $V(x)$  for some choice of  $\lambda$ 's This function has a unique minimum over all the state space at the origin. Moreover,  $V(x) \rightarrow \infty$  as  $k \times k \rightarrow \infty$ . Calculate the derivative of  $V$  along the trajectories of the system

$$V(x) = 2\lambda_1 x_1 (-x_1 + 2x_1^2x_2) + 2\lambda_2 x_2 (-x_2) = -2\lambda_1 x_1^2 - 2\lambda_2 x_2^2 + 4\lambda_1 x_1^3 x_2$$

**Theorem (1)** : let the origin  $x = 0 \in D \subset R^n$  be an equilibrium point for  $\dot{x} = f(x)$ . Let  $V : D \rightarrow R$  be a continuous differentiable function such that  $V(0) = 0$  and  $V(x) > 0, x \in D \setminus \{0\}$

$$\dot{V}(x) \leq 0 \quad \forall x \in D \quad (9)$$

Then  $x = 0$  is stable. Moreover, if  $\dot{V}(x) < 0 \quad \forall x \in D \setminus \{0\}$  Then  $x = 0$  asymptotically stable.

**Remark 1** If  $V(x) > 0, \forall x \in D \setminus \{0\}$ , then  $V$  is called locally positive definite. If  $V(x) > 0, \forall x \in D \setminus \{0\}$ , then  $V$  is called a Lyapunov function for the system  $\dot{x} = f(x)$ .

### Stability of nonautonomous Systems:

Consider the nonlinear autonomous system

$$\dot{x} = f(x) \quad (10)$$

Where  $f : D \rightarrow R^n$ , the domain  $D \subseteq R^n$  to  $R^n$  suppose the system (10) has an equilibrium point  $\bar{x} \in D$ , i.e.,  $f(\bar{x})$  if the equilibrium point  $\bar{x}$  is stable. In the sequel, we assume that  $\bar{x}$  is the origin of state space. This can be done without any loss of generality since we can always apply a change of variables to  $\xi = x - \bar{x}$  to obtain

$$\dot{\xi} = f(\xi + \bar{x})$$

and then study the stability of the new system with respect to  $\xi = 0$ , the origin. We have the following two types of stability.

**Definition (2)** : The equilibrium point  $x = 0$  of (10) is

1. **Stable**, if for each  $\epsilon > 0$  there exist  $\delta > 0$  such that  $\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall t > t_0$  (11)

2. **asymptotically stable**, if it is stable and in addition  $\delta$  can be chosen such that  $\|x(t_0)\| < \delta \Rightarrow \lim_{t \rightarrow \infty} \|x(t)\| = 0$ , (12)

Therefore, if  $\dot{V}(x)$  is negative,  $V$  will decrease along the solution of  $\dot{x} = f(x)$ . We are now ready to state Lyapunov's stability theorem.

### Lyapunov's Indirect Method:

We prove stability of the system by considering the properties of the linearization of the system. Before proving the main result, we require an intermediate result.

**Definition (3):** A matrix  $A \in \mathbb{R}^{n \times n}$  is called Hurwitz or asymptotically stable, if and only if  $\text{Re}(\lambda_i) < 0, \forall i = 1, 2, \dots, n$  where  $\lambda_i$ 's are the eigenvalues of the matrix  $F$ . Consider the system  $\dot{x} = Ax$ . We look for a quadratic function  $V(x) = x^T P x$  where  $P = P^T > 0$ . Then

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} = x^T (A^T P + P A x) = -x^T Q x$$

If there exists  $Q = Q^T > 0$  such that  $A^T P + P A = -Q$ , then  $V$  is a Lyapunov function and  $x = 0$  is globally stable. This equation is called the Matrix Lyapunov Equation.

**Theorem (2):** For  $A \in \mathbb{R}^{n \times n}$  the following statements are equivalent

1.  $A$  is Hurwitz
2. For all  $Q = Q^T > 0$  there exist unique  $P = P^T > 0$  satisfying the Lyapunov Equation.  $A^T P + P A = -Q$ ,

### DEFINITION OF DELAY DIFFERENTIAL EQUATIONS:

The general form of DDE system to be considered is then

$$\dot{x}(t) = f(t, x_t) \quad (13)$$

Where  $x: \mathbb{R} \rightarrow \mathbb{R}^N, f: \mathbb{R} \times C \rightarrow \mathbb{R}^N, x$  is a solution of (10) on  $[t_0 - r, t_0 + A]$  for some  $A > 0$  with initial condition  $\emptyset \in C$  if  $x \in C([t_0 - r, t_0 + A], \mathbb{R}^N), x(t)$  satisfies (13) for  $t \in [t_0, t_0 + A]$  and  $x_0 = \emptyset$  on  $[-r, 0]$ .

**Definition (4):** the solution  $x = X(t)$  of (10) is said to be

- stable if, given any  $\epsilon > 0$  and any  $t_0 \in \mathbb{R}$  there exist  $\delta = \delta(\epsilon, t_0)$  such that  $\|x_{t_0} - X_{t_0}\| < \delta \Rightarrow |x(t) - X(t)| < \epsilon, \forall t \geq t_0$  (14) for any solution  $x(t)$  of (13),
- uniformly stable if, for every  $\epsilon > 0$ , there exist  $\delta = \delta(\epsilon)$ , independent of  $t_0$ , such that (14) is satisfied for all  $t_0 \in \mathbb{R}$ .
- unstable if it is not stable.
- asymptotically stable if it is stable and for any  $t_0 \in \mathbb{R}$  there exist a positive constant  $c = c(t_0)$  such that  $\|x_{t_0} - X_{t_0}\| < c \Rightarrow x(t) - X(t) \rightarrow 0$  as  $x \rightarrow 0$  for any solution  $x(t)$  of (13).
- uniformly asymptotically stable if it is uniformly stable and there exist a positive constant  $c$ , independent of  $t_0$ , such that, for every  $\eta > 0$ , there exist  $T = T(\eta) > 0$  such that for all  $t_0 \in \mathbb{R}$   $\|x_{t_0} - X_{t_0}\| < c \Rightarrow |x(t) - X(t)| < \eta, \forall t > t_0 + T(\eta)$  for any solution  $x(t)$  of (13).
- globally uniformly asymptotically stable if it is uniformly stable with  $\delta(\epsilon)$  satisfying  $\lim_{\epsilon \rightarrow \infty} \delta(\epsilon) = \infty$ , and for all positive  $\eta$  and  $c$ , there exist  $T = T(\eta, c) > 0$  such that, for all  $t_0 \in \mathbb{R}$   $\|x_{t_0} - X_{t_0}\| < c \Rightarrow |x(t) - X(t)| < \eta, \forall t > t_0 + T(\eta, c)$ , for any solution  $x(t)$  of (13),

### Stability of delay differential equations

Delay differential equations (DDEs) are a type of differential equation in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times. DDEs are also called time-delay systems, systems with aftereffect or dead-time, hereditary systems, equations with deviating argument, or differential-difference equations. They belong to the class of systems with the functional state, i.e. partial differential equations (PDEs) which are infinite dimensional, as opposed to ordinary differential equations (ODEs) having a finite dimensional state vector. Four points may give a possible explanation of the popularity of DDEs.

(1) Aftereffect is an applied problem: it is well known that, together with the increasing expectations of dynamic performances, engineers need their models to behave more like the real process. Many processes include aftereffect phenomena in their inner dynamics. In addition, actuators, sensors, communication networks that are now involved in feedback control loops introduce such delays. Finally, besides actual delays, time lags are frequently used to simplify very high order models. Then, the interest for DDEs keeps on growing in all scientific areas and, especially, in control engineering.

(2) Delay systems are still resistant to many *classical* controllers: one could think that the simplest approach would consist in replacing them by some finite-dimensional approximations. Unfortunately, ignoring effects which are adequately represented by DDEs is not a general alternative: in the best situation (constant and known delays), it

leads to the same degree of complexity in the control design. In worst cases (time-varying delays, for instance), it is potentially disastrous in terms of stability and oscillations.

(3) Delay properties are also surprising since several studies have shown that voluntary introduction of delays can also benefit the control.

(4) In spite of their complexity, DDEs however often appear as simple infinite-dimensional models in the very complex area of partial differential equations

**Theorem (3) :**(Razumikhin Theorem). Let  $x = 0$  be a solution of (10). Suppose that  $f : \mathbb{R} \times C \rightarrow \mathbb{R}^n$  in (10) takes  $\mathbb{R} \times$  (bounded sets in  $C$ ) into bounded sets in  $\mathbb{R}^n$ , and  $u, v, w : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  are continuous non-decreasing functions, with  $u(s), v(s) > 0$  for all  $s > 0, u(0) = v(0) = 0$  and  $v$  strictly increasing. Suppose further that there exists a continuous function  $V : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$  such that:

- i.  $u(|x|) \leq V(t, x) \leq v(|x|), \forall t \in \mathbb{R}, \forall x \in \mathbb{R}^n$
- ii.  $\dot{V}(t, x(t)) \leq -w(|x(t)|)$  if  $V(t + \theta, x(t + \theta)) \leq V(t, x(t))$  for  $\theta \in [-r, 0]$  where  $x(t)$  is any trajectory of (10). Then the solution  $x = 0$  is uniformly stable.

**Theorem (4)** (Razumikhin theorem for uniform asymptotic stability): Suppose that all assumption of theorem 6 are satisfied and also  $w(s) > 0$  for  $s > 0$  that, in addition, there exists a continuous non-decreasing function  $q : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  satisfying  $q(s) > s$  for all  $s > 0$  such that it can be strengthened to

$$\dot{V}(t, x(t)) \leq -w(|x(t)|) \text{ if } V(t + \theta, x(t + \theta)) \leq q(V(t, x(t))) \tag{12}$$

for  $\theta \in [-r, 0]$  where  $x(t)$  is any trajectory of (10) then the solution  $x = 0$  is uniformly asymptotically stable. If further  $u(s) \rightarrow \infty$  as  $s \rightarrow \infty$ , then the solution  $x = 0$  is globally asymptotically stable.

### APPLICATION TO TIME VARYING WIRELESS NETWORKS (PROBLEM FORMULATION)

We consider a wireless system consisting of  $N$  users. Let the transmitted power from the antenna of user  $i$  at time  $t$  be given by  $p_i(t)$ , and define  $p = (p_1, p_2, \dots, p_N)^T$ . Let the link gain between the transmitter of user  $j$  and the receiver of user  $i$  be  $G_{ij}$  and the background noise in the power transmitted at user  $i$  be  $v_i$ . We may then write the following expression for the effective interference at the receiver of user  $i$ ,

$$R_i(p) = \frac{1}{G_{ii}} \left( \sum_{j \neq i} G_{ij} p_j + v_i \right)$$

we define the signal-to-interference-ratio (SIR) at user  $i$  as  $\Gamma_i(p) = \frac{p_i}{R_i(p)}$ . The continuous form of the Foschini–

Miljanic algorithm

$$\frac{dp_i(t)}{dt} = k_i(-p_i(t) + \gamma_i R_i(p)), \tag{13}$$

where  $k_i$  is a positive constant representing the aggressiveness of the feedback in the system and  $\gamma_i$  is a positive constant representing the target SIR value for node  $i$ . In particular, we consider the system

$$\frac{dp_i(t)}{dt} = k_i(-p_i(t) + I_i(t, p)) \tag{14}$$

where  $I(t, p) = (I_1(t, p), I_2(t, p), \dots, I_N(t, p))^T$  is required to satisfy the following two properties, motivated by the properties of the original form of the interference term, at all times  $t$  for all  $p \geq 0$ :

- i. Monotonicity: if  $p \geq p'$ , then  $I(t, p) \geq I(t, p')$ ,
- ii. Scalability: there exists a continuous function  $\delta : (1, \infty) \rightarrow \mathbb{R}^+$  such that, for any  $\alpha > 1, I_i(t, p) - \frac{1}{\alpha} I_i(t, \alpha p) \geq \delta(\alpha)$  for all  $i \in \{1, 2, \dots, N\}$ .

### CONCLUSIONS

We begin by studying (11) in the absence of delays, before introducing into the framework delays which may be heterogeneous and time-dependent. In particular, we show in the undelayed case that if a bounded solution  $p = P(t)$  exists then this is uniformly asymptotically stable. For the delayed case we show that if a solution  $p = P(t)$  exists for which the delayed generalised nonlinearity  $I$  is bounded, then this is also uniformly asymptotically stable. In both cases the stability is also shown to be global, i.e. for all initial conditions all solutions  $p(t)$  have the same asymptotic behavior.

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